7 On the Quantum Theory of Radiation[†]

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THE formal similarity of the curve of the chromatic distribution of black-body radiation and the Maxwell velocity-distribution is too striking to be hidden for long. Indeed, already Wien in his important theoretical paper in which he derived his displacement law

$$\rho = \nu^3 f(\nu/T) \tag{1}$$

was led by this similarity to a further determination of the radiation formula. It is well known that he then found the formula

$$\rho = \alpha\nu^3 \, e^{-h\nu/kT}, \tag{2}$$

which is also nowadays accepted as being correct as a limiting law for large values of ν/T (Wien's radiation law). We know nowadays that no considerations based on classical mechanics and electrodynamics can give us a usable radiation formula, and that classical theory necessarily leads to the Rayleigh formula

$$\rho = \frac{k\alpha}{h} \nu^2 T. \tag{3}$$

As soon as Planck in his classical investigation based his radiation formula

$$\rho = \alpha\nu^3 \frac{1}{e^{h\nu/kT} - 1} \tag{4}$$

on the assumption of discrete elements of energy, from which very

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quickly quantum theory developed, it was natural that Wien's discussion which led to equation (2) became forgotten.

Recently[1][†] I found a derivation of Planck's radiation formula which is based upon the basic assumption of quantum theory and which is related to Wien's original considerations; in this derivation, the relationship between the Maxwell distribution and the chromatic black-body distribution plays a role. This derivation is of interest not only because it is simple, but especially because it seems to clarify somewhat the at present unexplained phenomena of emission and absorption of radiation by matter. I have shown, on the basis of a few assumptions about the emission and absorption of radiation by molecules, which are closely related to quantum theory, that molecules distributed in temperature equilibrium over states in a way which is compatible with quantum theory are in dynamic equilibrium with the Planck radiation. In this way, I deduced in a remarkably simple and general manner Planck's formula (4). It was a consequence of the condition that the distribution of the molecules over the states of their internal energy, which is required by quantum theory, must be established solely through the absorption and emission of radiation.

If the assumptions about the interaction between radiation and matter which we have introduced are essentially correct, they must, however, yield more than the correct statistical distribution of the internal energy of the molecules. In fact, in absorption and emission of radiation, momentum is transferred to the molecules; this entails that merely through the interaction of radiation and molecules the velocities of the molecules will acquire a certain distribution. This must clearly be the same velocity distribution as the one which the molecules attain through the action of their mutual collisions alone, that is, it must be the same as the Maxwell distribution. We must require that the average kinetic energy (per degree of freedom) which a molecule acquires in the Planck radiation field of temperature T is equal to $\frac{1}{2}kT$; this must be true independent of the nature of the molecules considered and independent of the frequencies of the light emitted or absorbed by them. In the present paper, we want to show that our simple hypotheses about the elementary processes of emission and absorption obtain another support.

In order to obtain the above-mentioned result we must, however, complete to some extent the hypotheses upon which our earlier work was based, as the earlier hypotheses were concerned only with the exchange of energy. The question arises: does the molecule receive an impulse when it absorbs or emits the energy ε? Let us, for instance, consider the emission from the point of view of classical electrodynamics. If a body emits the energy ε, it receives a recoil (momentum) ε/c if all of the radiation ε is emitted in the same direction. If, however, the emission takes place as an isotropic process, for instance, in the form of spherical waves, no recoil at all occurs. This alternative also plays a role in the quantum theory of radiation. When a molecule during a transition from one quantum-theoretically possible state to another absorbs or emits energy ε in the form of radiation, such an elementary process can be thought of either as being a partially or completely directed or as being a symmetrical (non-directional) process. *It now turns out that we arrive at a consistent theory only, if we assume each elementary process to be completely directional.* This is the main result of the following considerations.

1. Basic Hypothesis of Quantum Theory. Canonical Distribution over States

According to quantum theory, a molecule of a given kind can take up—apart from its orientation and its translational motion—only a discrete set of states $Z_1, Z_2, \ldots, Z_n, \ldots$ with (internal) energies $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n, \ldots$. If molecules of this kind form a gas of temperature T, the relative occurrence W_n of these states Z_n is given by the formula giving the canonical distribution of statistical mechanics:

$$W_n = p_n e^{-\varepsilon_n/kT}. \tag{5}$$

In this equation $k = R/N$ is the well-known Boltzmann constant, and p_n a number which is characteristic for the molecule and its

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[†] The considerations given in that paper are repeated in the present one.

*n*th quantum state and which is independent of T; it can be called the statistical "weight" of the state. One can derive equation (5) either from Boltzmann's principle or by purely thermodynamic means. Equation (5) expresses the greatest generalisation of Maxwell's velocity distribution law.

Recent important progress in quantum theory relates to the theoretical determination of quantum theoretically possible states Z_n and their weight p_n. For our considerations of the principles involved in radiation, we do not need a detailed determination of the quantum states.

2. Hypotheses about Energy Exchange through Radiation

Let Z_n and Z_m be two quantum-theoretically possible states of the gas molecule, and let their energies ε_n and ε_m satisfy the inequality $\varepsilon_m > \varepsilon_n$. Let the molecule be able to make a transition from the state Z_n to the state Z_m by absorbing radiative energy $\varepsilon_m - \varepsilon_n$; similarly let a transition from Z_m to Z_n be possible in which this radiative energy is emitted. Let the frequency of the radiation absorbed or emitted by the molecule in such transitions be ν; it is characteristic for the combination (m, n) of the indices.

We make a few hypotheses about the laws valid for this transition; these are obtained by using the relations known from classical theory for a Planck resonator, as the quantum-theoretical relations which are as yet unknown.

(a) *Spontaneous emission.*† It is well known that a vibrating Planck resonator emits according to Hertz energy independent of whether it is excited by an external field or not. Accordingly, let it be possible for a molecule to make without external stimulation a transition from the state Z_m to the state Z_n while emitting the radiation energy $\varepsilon_m - \varepsilon_n$ of frequency ν. Let the probability dW that this will in fact take place in the time interval dt be

$$dW = A_m^n \, dt, \tag{A}$$

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† Einstein uses *Ausstrahlung* and *Einstrahlung* for spontaneous emission and induced radiation [D. t. H.].

where A_m^n denotes a constant which is characteristic for the combination of indices considered.

The statistical law assumed here corresponds to the law of a radioactive reaction, and the elementary process assumed here corresponds to a reaction in which only γ-rays are emitted. It is not necessary to assume that this process takes place instantaneously; it is only necessary that the time this process takes is negligible compared with the time during which the molecule is in the state Z_1, ...

(b) *Induced radiation processes.* If a Planck resonator is in a radiation field, the energy of the resonator can be changed by the transfer of energy from the electromagnetic field to the resonator; this energy can be positive or negative depending on the phases of the resonator and of the oscillating field. Accordingly we introduce the following quantum-theoretical hypothesis. Under the influence of a radiation density ρ of frequency ν a molecule can make a transition from the state Z_n to the state Z_m by absorbing the radiative energy $\varepsilon_m - \varepsilon_n$ and the probability law for this process is

$$dW = B_n^m \rho \, dt. \tag{B}$$

Similarly, a transition $Z_m \rightarrow Z_n$ may also be possible under the influence of the radiation; in this process the radiative energy $\varepsilon_m - \varepsilon_n$ will be freed according to the probability law

$$dW = B_m^n \rho \, dt. \tag{B'}$$

The B_n^m and B_m^n are constants. These two processes we shall call "changes in state, induced by radiation".

The question now arises: what is the momentum transferred to the molecule in these changes in state? Let us begin with the induced processes. If a radiation beam with a well-defined direction does work on a Planck resonator, the corresponding energy is taken from the beam. According to the law of conservation of momentum, this energy transfer corresponds also to a momentum transfer from the beam to the resonator. The resonator is thus subject to the action of a force in the direction of the beam. If the energy transferred is negative, the action of the force on the resonator is also in the opposite direction. This means clearly the following in the case of the quantum hypothesis. If through the irradiation by a beam of light a transition $Z_n \rightarrow Z_m$ is induced, the momentum $(\varepsilon_m - \varepsilon_n)/c$ is transferred to the molecule in the direction of propagation of the beam. In the induced transition $Z_m \rightarrow Z_n$ the transferred momentum has the same magnitude but is in the opposite direction. We assume that in the case where the molecule is simultaneously subjected to several radiation beams, the total energy $\varepsilon_m - \varepsilon_n$ of an elementary process is absorbed from or added to *one* of these beams, so that also in that case the momentum $(\varepsilon_m - \varepsilon_n)/c$ is transferred to the molecule.

In the case of a Planck resonator, when the energy is emitted through a spontaneous emission process, no momentum is transferred to the resonator, since according to classical theory the emission is in the form of a spherical wave. We have, however, already noted that we can only obtain a consistent quantum theory by assuming that the spontaneous emission process is also a directed one. In that case, in each spontaneous emission elementary process $(Z_m \rightarrow Z_n)$ momentum of magnitude $(\varepsilon_m - \varepsilon_n)/c$ is transferred to the molecule. If the molecule is isotropic, we must assume that all directions of emission are equally probable. If the molecule is not isotropic, we arrive at the same statement if the orientation changes in a random fashion in time. We must, of course, make a similar assumption for the statistical laws (B) and (B′) for the induced processes, as otherwise the constants should depend on direction, but we can avoid this through the assumption of isotropy or pseudo-isotropy (through time-averaging) of the molecule.

3. Derivation of the Planck Radiation Law

We now ask for that radiation density ρ which must be present in order that the exchange of energy between radiation and molecules according to the statistical laws (A), (B), and (B′) does

not perturb the distribution (5) of the molecules. For this it is necessary and sufficient that on the average per unit time as many elementary processes of type (B) take place as of types (A) and (B′) combined. This combination leads, because of (5), (A), (B), and (B′), to the following equation for the elementary processes corresponding to the index combination (m, n):

$$p_n e^{-\varepsilon_n/kT} B_n^m \rho = p_m e^{-\varepsilon_m/kT} (B_m^n \rho + A_m^n).$$

If, furthermore, ρ will increase to infinity with T, as we shall assume, the following relation must exist between the constants B_n^m and B_m^n:

$$p_n B_n^m = p_m B_m^n. \tag{6}$$

We then obtain from our equation the following condition for dynamic equilibrium:

$$\rho = \frac{A_m^n / B_m^n}{e^{(\varepsilon_m - \varepsilon_n)/kT} - 1}. \tag{7}$$

This is the temperature-dependence of the radiation density of the Planck law. From Wien's displacement law (1) it follows from this immediately that

$$\frac{A_m^n}{B_m^n} = \alpha \nu^3, \tag{8}$$

and

$$\varepsilon_m - \varepsilon_n = h\nu, \tag{9}$$

where α and h are constants. To find the numerical value of the constant α, we should have an exact theory of the electrodynamic and mechanic processes; for the time being we must use the Rayleigh limit of high temperatures, for which the classical theory is valid as a limiting case.

Equation (9) is, of course, the second main hypothesis of Bohr's theory of spectra of which we can now state after Sommerfeld's and Epstein's extensions that it belongs to those parts of our science which are sure. It contains implicitly, as I have shown, also the photochemical equivalence rule.

4. Method of Calculating the Motion of Molecules in the Radiation Field

We now turn to the investigation of the motions which our molecules execute under the influence of the radiation. To do this, we use a method which is well known from the theory of Brownian motion, and which I have used already many times for numerical calculations of motion in radiation. To simplify the calculations, we only perform them for the case where the motion takes place only in one direction, the X-direction of our system or coordinates. We shall moreover restrict ourselves to calculating the average value of the kinetic energy of the translational motion, and thus do not give the proof that these velocities v are distributed according to Maxwell's law. Let the mass M of the molecule be sufficiently large that we can neglect higher powers of v/c in comparison with lower ones; we can then apply ordinary mechanics to the molecule. Moreover, without any real loss of generality, we can perform our calculations as if the states with indices m and n were the only ones which the molecule can take on.

The momentum Mv of a molecule is changed in two ways in the short time τ. Although the radiation is the same in all directions, because of its motion the molecule will feel a force acting in the opposite direction of its motion which comes from the radiation. Let this force be Rv, where R is a constant to be evaluated later on. This force would bring the molecule to rest, if the irregularity of the action of the radiation did not have as a consequence that during the time τ a momentum Δ of varying sign and varying magnitude is transferred to the molecule; this unsystematic influence will against the earlier mentioned force maintain a certain motion of the molecule. At the end of the short time τ, which we are considering, the momentum of the molecule will have the value

$$Mv - Rv\tau + \Delta.$$

As the velocity distribution must remain the same in time, this quantity must have the same average absolute magnitude as Mv; therefore, the average squares of those two quantities, averaged over a long period or over a large number of molecules, must be equal to one another:

$$\overline{(Mv - Rv\tau + \Delta)^2} = \overline{(Mv)^2}.$$

As we have separately taken into account the systematic influence of v on the momentum of the molecule, we must neglect the average $\overline{\Delta v}$. Expanding the left-hand side of the equation, we get then

$$\overline{\Delta^2} = 2RM\overline{v^2}\tau. \qquad (10)$$

The average $\overline{v^2}$ for our molecules, which is caused by radiation of temperature T through its interaction with the molecules must be equal to the average value $\overline{v^2}$, which according to the kinetic theory of gases a molecule in the gas would have according to the gas laws at the temperature T. Otherwise, the presence of our molecules would disturb the thermodynamic equilibrium between black-body radiation and any gas of the same temperature. We must then have

$$\tfrac{1}{2}M\overline{v^2} = \tfrac{1}{2}kT. \qquad (11)$$

Equation (10) thus becomes

$$\frac{\overline{\Delta^2}}{\tau} = 2RkT. \qquad (12)$$

The investigation must now proceed as follows. For given radiation $[\rho(\nu)]$ we can calculate $\overline{\Delta^2}$ and R with our hypotheses about the interaction between radiation and molecules. Inserting these results into (12), this equation must be satisfied identically if ρ as function of ν and T is expressed by the Planck equation (4).

5. Calculation of R

Let a molecule of the kind considered move uniformly with velocity v along the X-axis of the system of coordinates K. We ask for the average momentum transferred by the radiation to the molecule per unit time. To be able to evaluate this, we must consider the radiation in a system of coordinates K' which is at rest relative to the molecule under consideration, because we have only formulated our hypotheses about emission and absorption for molecules at rest. The transformation to the system K' has often been given in the literature and especially accurately in Mosengeil's Berlin thesis.[2] For the sake of completeness, I shall, however, repeat the simple considerations.

In K the radiation is isotropic, that is, we have for the radiation per unit volume in a frequency range $d\nu$ and propagating in a direction within a given infinitesimal solid angle $d\kappa$:

$$\rho\, d\nu \frac{d\kappa}{4\pi}, \tag{13}$$

where ρ depends only on the frequency ν, but not on the direction. This particular radiation corresponds in the coordinate system K' to a particular radiation, which is also characterised by a frequency range $d\nu'$ and a certain solid angle $d\kappa'$. The volume density of this particular radiation is

$$\rho'(\nu', \phi')\, d\nu' \frac{d\kappa'}{4\pi}. \tag{13'}$$

This defines ρ'. It depends on the direction which is defined in the usual way by the angle ϕ' with the X'-axis and the angle ψ' between the projection in the $Y'Z'$-plane with the Y'-axis. These angles correspond to the angles ϕ and ψ which in a similar manner fix the direction of $d\kappa$ with respect to K.

First of all it is clear that the same transformation law must be valid between (13) and (13′) as between the squares of the amplitude A^2 and A'^2 of a plane wave of the appropriate direction of propagation. Therefore in the approximation we want, we have

$$\frac{\rho'(\nu', \phi')\, d\nu'\, d\kappa'}{\rho(\nu)\, d\nu\, d\kappa} = 1 - 2\frac{v}{c}\cos\phi, \tag{14}$$

or

$$\rho'(\nu', \phi') = \rho(\nu) \frac{d\nu}{d\nu'} \frac{d\kappa}{d\kappa'} \left(1 - 2\frac{v}{c}\cos\phi\right). \tag{14'}$$

The theory of relativity further gives the following formulae, valid in the approximation needed here,

$$\nu' = \nu\left(1 - \frac{v}{c}\cos\phi\right), \tag{15}$$

$$\cos\phi' = \cos\phi - \frac{v}{c} + \frac{v}{c}\cos^2\phi, \tag{16}$$

$$\psi' = \psi. \tag{17}$$

From (15) in the same approximation it follows that

$$\nu = \nu'\left(1 + \frac{v}{c}\cos\phi'\right).$$

Therefore, also in the same approximation,

$$\rho(\nu) = \rho\left(\nu' + \frac{v}{c}\nu'\cos\phi'\right),$$

or

$$\rho(\nu) = \rho(\nu') + \frac{\partial\rho(\nu')}{\partial\nu} \frac{v}{c} \nu' \cos\phi'. \tag{18}$$

Furthermore from (15), (16), and (17) we have

$$\frac{d\nu}{d\nu'} = 1 + \frac{v}{c}\cos\phi',$$

$$\frac{d\kappa}{d\kappa'} = \frac{\sin\phi\, d\phi\, d\psi}{\sin\phi'\, d\phi'\, d\psi'} = \frac{d(\cos\phi)}{d(\cos\phi')} = 1 - 2\frac{v}{c}\cos\phi'.$$

Using these two relations and (18), we get from (14′)

$$\rho(\nu', \phi') = \left[\rho(\nu) + \frac{v}{c}\nu'\cos\phi' \frac{\partial\rho(\nu)}{\partial\nu}\right]\left(1 - 3\frac{v}{c}\cos\phi'\right). \tag{19}$$

From (19) and our hypothesis about the spontaneous emission and the induced processes of the molecule, we can easily calculate the average momentum transferred per unit time to the molecule. Before doing this we must, however, say something to justify the method used. One can object that the equations (14), (15), and (16) are based upon Maxwell's theory of the electromagnetic field which is incompatible with quantum theory. This objection, however, touches the form rather than the essence of the matter. Whatever the form of the theory of electromagnetic processes, surely in any case the Doppler principle and the aberration law will remain valid, and thus also equations (15) and (16). Furthermore, the validity of the energy relation (14) certainly extends beyond that of the wave theory; this transformation law is, for instance, also valid according to the theory of relativity for the energy density of a mass having an infinitesimal rest mass and moving with (quasi-) light-velocity. We can thus claim the validity of equation (19) for any theory of radiation.

The radiation corresponding to the spatial angle $d\kappa'$ will according to (B) lead per second to

$$B_n^m \rho'(\nu',\phi')\frac{d\kappa'}{4\pi}$$

induced elementary processes of the type $Z_n \to Z_m$, provided the molecule is brought back to the state Z_n immediately after each such elementary process. In reality, however, the time spent per second in the state Z_n is according to (5) equal to

$$\frac{1}{S}p_n e^{-\varepsilon_n/kT},$$

where we used the abbreviation

$$S = p_n e^{-\varepsilon_n/kT} + p_m e^{-\varepsilon_m/kT}. \tag{20}$$

In actual fact the number of these processes per second is thus

$$\frac{1}{S}p_n e^{-\varepsilon_n/kT} B_n^m \rho'(\nu',\phi')\frac{d\kappa'}{4\pi}.$$

In each process the momentum

$$\frac{\varepsilon_m-\varepsilon_n}{c}\cos\phi'$$

is transferred to the molecule in the direction of the positive X'-axis. Similarly, we find, using (B') that the corresponding number of induced elementary processes of the kind $Z_m \to Z_n$ per second is

$$\frac{1}{S}p_m e^{-\varepsilon_m/kT} B_m^n \rho'(\nu',\phi')\frac{d\kappa'}{4\pi},$$

and in each such elementary process the momentum

$$-\frac{\varepsilon_m-\varepsilon_n}{c}\cos\phi'$$

is transferred to the molecule. The total momentum transferred per unit time to the molecule through induced processes is thus, taking (6) and (9) into account,

$$\frac{h\nu'}{cS}p_n B_n^m (e^{-\varepsilon_n/kT}-e^{-\varepsilon_m/kT})\int \rho'(\nu',\phi')\cos\phi'\frac{d\kappa'}{4\pi},$$

where the integration is over all elements of solid angle. Performing the integration we get, using (19), the value

$$-\frac{h\nu}{c^2 S}\left(\rho-\tfrac{1}{3}\nu\frac{\partial\rho}{\partial\nu}\right)p_n B_n^m (e^{-\varepsilon_n/kT}-e^{-\varepsilon_m/kT})\,v.$$

Here we have denoted the frequency involved by ν (instead of ν').

This expression represents, however, the total average momentum transferred per unit time to a molecule moving with a velocity v; because it is clear that the spontaneous emission processes which take place without the action of radiation do not have a preferential direction, considered in the system K', so that they can on average not transfer any momentum to the molecule. We obtain thus as the final result of our considerations:

$$R = \frac{h\nu}{c^2 S}\left(\rho-\tfrac{1}{3}\nu\frac{\partial\rho}{\partial\nu}\right)p_n B_n^m e^{-\varepsilon_n/kT}(1-e^{-h\nu/kT}). \tag{21}$$

6. Calculation of $\overline{\Delta^2}$

It is much simpler to calculate the influence of the irregularity of the elementary processes on the mechanical behaviour of the molecule, as we can base this calculation on a molecule at rest in the approximation which we have used from the start.

Let some event lead to the transfer of a momentum λ in the X-direction to a molecule. Let this momentum have varying sign and varying magnitude in different cases, but let there be such a statistical law for λ that the average value of λ vanishes. Let now $\lambda_1, \lambda_2, \ldots$ be the values of the momentum in the X-direction transferred to the molecule through several, independently acting causes so that the resultant transfer of momentum Δ is given by

$$\Delta = \Sigma \lambda_v.$$

As the average value $\overline{\lambda_v}$ vanishes for the separate λ_v, we must have

$$\overline{\Delta^2} = \Sigma \overline{\lambda_v^2}. \tag{22}$$

If the averages $\overline{\lambda_v^2}$ of the separate momenta are equal to one another $(=\overline{\lambda^2})$, and if l is the total number of momentum transferring processes, we have the relation

$$\overline{\Delta^2} = l\overline{\lambda^2}. \tag{22a}$$

According to our hypothesis in each elementary process, induced or spontaneous, the momentum

$$\lambda = \frac{h\nu}{c}\cos\phi$$

is transferred to the molecule. Here ϕ is the angle between the X-axis and a direction chosen randomly. Therefore we have

$$\overline{\lambda^2} = \frac{1}{3}\left(\frac{h\nu}{c}\right)^2. \tag{23}$$

As we assume that we may take all elementary processes which take place to be independent of one another, we may apply (22a). In that case, l is the number of all elementary processes taking place during the time τ. This is twice the number of the number of induced processes $Z_n \to Z_m$ during the time τ. We have thus

$$l = \frac{2}{S} p_n B_n^m e^{-\varepsilon_n/kT} \rho\tau. \tag{24}$$

We get from (23), (24), and (22)

$$\frac{\overline{\Delta^2}}{\tau} = \frac{2}{3S}\left(\frac{h\nu}{c}\right)^2 p_n B_n^m e^{-\varepsilon_n/kT} \rho. \tag{25}$$

7. Results

To prove now that the momenta transferred by the radiation to the molecules in accordance with our hypotheses never disturb the thermodynamic equilibrium, we only need to substitute the values (25) and (21) for $\overline{\Delta^2}/\tau$ and R which we have calculated after we have used (4) to replace in (21) the quantity

$$\left(\rho - \tfrac{1}{3}\nu\frac{\partial\rho}{\partial\nu}\right)(1 - e^{-h\nu/kT})$$

by $\rho h\nu/3kT$. It then turns out immediately that our fundamental equation (12) is identically satisfied.

The considerations which are now finished give strong support for the hypotheses given in Section 2 about the interaction between matter and radiation through absorption and emission processes, that is, through spontaneous and induced radiation processes. I was led to these hypotheses by my attempt to postulate as simply as possible a quantum theoretical behaviour of the molecules which would be similar to the behaviour of a Planck resonator of the classical theory. I obtained then in a natural fashion from the general quantum assumption for matter the second Bohr rule (equation (9)) as well as the Planck radiation formula.

The most important result seems to me, however, to be the one about the momentum transferred to the molecule in spontaneous or induced radiation processes. If one of our assumptions about this momentum transfer is changed, this would lead to a violation of equation (12); it seems hardly possible to remain in agreement with this relation which is required by the theory of heat otherwise than on the basis of our assumptions. We can thus consider the following as rather certainly proved.

If a ray of light causes a molecule hit by it to absorb or emit through an elementary process an amount of energy $h\nu$ in the form of radiation (induced radiation process), the momentum $h\nu/c$ is always transferred to the molecule, and in such a way that the momentum is directed along the direction of propagation of the ray if the energy is absorbed, and directed in the opposite direction, if the energy is emitted. If the molecule is subjected to the action of several directed rays of light, always only one of them will participate in an induced elementary process; this ray alone defines then the direction of the momentum transferred to the molecule.

If the molecule undergoes a loss of energy of magnitude $h\nu$ without external influence, by emitting this energy in the form of radiation (spontaneous emission), this process is also a *directed one*. There is no emission in spherical waves. The molecule suffers in the spontaneous elementary process a recoil of magnitude $h\nu/c$ in a direction which is in the present state of the theory determined only by "chance".

These properties of the elementary processes required by equation (12) make it seem practically unavoidable that one must construct an essentially quantum theoretical theory of radiation. The weakness of the theory lies, on the one hand, in the fact that it does not bring any nearer the connexion with the wave theory and, on the other hand, in the fact that it leaves moment and direction of the elementary processes to "chance"; all the same, I have complete confidence in the reliability of the method used here.

Still one more general remark may be made here. Practically all theories of black-body radiation are based on a consideration of the interaction between radiation and molecules. However, in general one restricts oneself to considering energy-exchange, without taking momentum-exchange into account. One feels easily justified in this as the smallness of the momenta transferred by the radiation entails that these momenta are practically always in reality negligible compared to other processes causing a change in motion. However, for the *theoretical* discussion, these small actions must be considered to be completely as important as the obvious actions of the *energy*-exchange through radiation, as energy and momentum are closely connected; one can, therefore, consider a theory to be justified only when it is shown that according to it the momenta transferred by the radiation to the matter lead to such motion as is required by the theory of heat.

References

1. A. Einstein, *Verh. Dtsch. Phys. Ges.* **18**, 318 (1916).
2. K. von Mosengeil, *Ann. Physik* **22**, 867 (1907).